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Particle Creation in the Oscillatory Phase of Inflaton

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A thermal squeezed state representation of inflaton is constructed for a flat Friedmann–Robertson–Walker (FRW) background metric and the phenomenon of particle creation is examined during the oscillatory phase of inflaton, in the semiclassical theory of gravity. An approximate solution to the semiclassical Einstein equation is obtained in thermal squeezed state formalism perturbatively and is found obey the same power-law expansion as that of classical Einstein equation. In addition to that the solution shows oscillatory in nature except on a particular condition. It is also noted that, the coherently oscillating nonclassical inflaton, in thermal squeezed vacuum state, thermal squeezed state, and thermal coherent state, suffers particle production and the created particles exhibit oscillatory behavior. The present study can account for the postinflation particle creation due to thermal and quantum effects of inflation in a flat FRW universe.

KEY WORDS: particle creation; inflaton; thermal squeezed states; thermal coherent states.

1. INTRODUCTION

According to the simplest version of the inflationary scenario, the universe in the past expanded exponentially with time, while its energy density was dominated by the effective potential energy density of a scalar field, called the inflaton. Sooner or later, inflation terminated and the inflaton field started quasiperiodic motion with slowly decreasing amplitude. The universe was empty of particles after inflation and particles of various kinds created due to the quasiperiodic evolution of the inflaton field. The universe became hot again due the oscillations and decay of the created particles of various kinds. Form then on, it can be described by the hot big bang theory.

The standard cosmology provides reliable and tested account of the history of the universe from about 0.01 s after the big bang until today, some 15 billion years later. Despite its success, the hot big bang model left many features of the universe unexplained. The most important of these are horizon problem,

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singularity problem, flatness problem, homogeneity problem, structure formation problem, monopole problem, and so on. All these problems are very difficult and defy solution within the standard cosmology. Most of these problems have been either completely resolved or considerably relaxed in the context of inflationary scenario (Guth, 1981). At present there are different versions (Albrecht and Steinhardt, 1982; Brandenberger, 1985; Linde, 1982) of the inflationary scenario. The main feature of all these versions is known as the inflationary paradigm. Inflationary cosmology is also widely accepted because of its success in explaining cosmological observations (Liddle and Lyth, 2000).

Most of the inflationary scenarios are based on the classical gravity of the Friedmann equation and the scalar field equation in the Friedmann–Robertson–Walker (FRW) universe, assuming its validity even at the very early stage of the universe. However, quantum effects of matter fields and quantum fluctuations are expected to play a significant role in this regime, though quantum gravity effects are still negligible. Therefore, the proper description of a cosmological model can be studied in terms of the semiclassical gravity of the Friedmann equation with quantized matter fields as the source of gravity. The semiclassical quantum gravity seems to be a viable method throughout the whole nonequilibrium quantum process from the preinflation period of hot plasma in thermal equilibrium to the inflation period and finally to the matter-dominated period.

Recently, the study of quantum properties of inflaton has received much attention in semiclassical theory of gravity and inflationary scenarios (Dongsu *et al.*, 1998; Kim and Page, 1999). In the new inflation scenario (Guth and Pi, 1995) quantum effects of the inflaton were partially taken into account by using one-loop effective potential and an initial thermal condition. In the stochastic inflation scenario (Linde *et al.*, 1994) the inflaton was studied quantum mechanically by dealing with the phase–space quantum distribution function and the probability distribution (Habib, 1992). The aforementioned studies show that results obtained in classical gravity are quite different from those in semiclassical gravity. Such studies reveal that quantum effects and quantum phenomena play an important role in inflation scenario and the related issues. Recently, it has been found that nonclassical state formalisms are quite useful to deal with quantum effects in cosmology (Albrecht *et al.*, 1994; Berger, 1981; Brandenberger *et al.*, 1992; Gasperini and Giovanni, 1994; Grishchuk and Sidorov, 1993; Hu *et al.*, 1994; Kuo and Ford, 1993; Matacz *et al.*, 1993), particularly squeezed states and coherent state formalism of quantum optics (Schumaker, 1986).

The above mentioned squeezed states and coherent state formalisms are zero temperature states. There exist a thermal counterparts of coherent and squeezed states and are useful to deal with finite temperature effects and quantum effects. From the cosmological point of view it would be more natural to consider the temperature effects on the background of FRW metric. Therefore, this motivates the study of thermal squeezed states and thermal coherent states in cosmology.

The goal of the present paper is to study quantum and finite temperature effects of minimally coupled massive inflaton in the FRW universe. Hence examined the thermal and quantum particle creation, in the oscillatory phase, of the inflaton in thermal coherent and thermal squeezed state formalisms, in the semiclassical theory of gravity. For the present study we follow the unit system $c = G = \hbar = 1$.

2. THERMAL SQUEEZED STATES AND THERMAL COHERENT STATES

The thermo field dynamic (Umezawa, 1993) formalism can be use to get the thermal counterparts of coherent and squeezed states. The main feature of thermo field dynamics is the thermal Bogoliubov transformation that maps the theory from zero to finite temperature. One can construct a thermal vacuum $|0(\beta)\rangle$ annihilated by thermal annihilation operators and can express the average value of any observable A as the expectation value in the thermal vacuum (Umezawa, 1993)

$$Z(\beta)^{-1}\mathrm{tr}[\rho A] = \langle 0(\beta)|A|0(\beta)\rangle, \tag{1}$$

where ρ is the distribution function, $\beta = \frac{1}{kT}$ and k Boltzmann's constant, and T the temperature. In order to fulfill the requirement (1), the vacuum should belong to the direct space between the original Fock space by an identical copy of it denoted by a tilde. Therefore,

$$|0(\beta)\rangle = e^{-iM}|0, \tilde{0}\rangle, \quad M = -i\theta(\beta)(a^\dagger\tilde{a}^\dagger - a\tilde{a}), \tag{2}$$

where $a, a^\dagger$ are the annihilation and creation operators in original Fock space and $\tilde{a}, \tilde{a}^\dagger$ are the same for the tilde space, and are obeying boson commutation relations $[a, a^\dagger] = [\tilde{a}, \tilde{a}^\dagger] = 1$, the other combinations are zero.

The density matrix approach usually gives us a convenient method for incorporating finite temperature effects. Hence, various definitions of thermal coherent states (tcs) can be summarized by giving its density matrix and it can be written for the single mode case as (Ezawa *et al.*, 1991)

$$\rho_{\mathrm{tcs}} = D^\dagger(\alpha)e^{-\beta\omega a^\dagger a}D(\alpha), \tag{3}$$

where α is a complex number specifying the coherent state, ω is the energy of the mode, and

$$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a). \tag{4}$$

The characteristic function for single mode thermal coherent state, $\mathcal{Q}_{\mathrm{tcs}}$, is defined by (Ezawa *et al.*, 1991)

$$\mathcal{Q}_{\mathrm{tcs}}(\eta, \eta^*) = \exp[-f(\beta)|\eta|^2 + \eta^*\alpha - \eta\alpha^*], \tag{5}$$

where η and η^* are as independent variables and,

$$f(\beta) = \frac{1}{e^{\beta\omega} - 1}. \tag{6}$$

Similarly the density matrix for a single mode thermal squeezed states (tss) is given by (Ezawa *et al.*, 1991)

$$\rho_{\text{tss}} = D^\dagger(\alpha) S^\dagger(\xi) e^{-\beta a^\dagger a} S(\xi) D(\alpha), \tag{7}$$

where

$$S(\xi) = \exp[(\xi a^{\dagger^2} - \xi^* a^2)/2], \quad \xi = r e^{i\vartheta}. \tag{8}$$

Here r is the squeezing parameter and ϑ is the squeezing angle.

The characteristic function of a single mode thermal squeezed state is given by

$$\begin{aligned} \mathcal{Q}_{\text{tss}}(\eta, \eta^*) = \exp\bigg[&-|\eta|^2 \left(\sinh^2 r \coth \frac{\beta\omega}{2} + f(\beta) \right) \\ &- \frac{\cosh r \sinh r}{2} \coth \frac{\beta\omega}{2} (e^{-i\varphi}\eta^2 + e^{i\varphi}\eta^{*2}) - \eta\alpha^* + \eta^*\alpha \bigg]. \end{aligned} \tag{9}$$

The density matrix for a single mode thermal squeezed vacuum (tsv) is given by

$$\rho_{\text{tsv}} = S^\dagger(\xi) e^{-\beta a^\dagger a} S(\xi), \tag{10}$$

and the characteristic function is

$$\begin{aligned} \mathcal{Q}_{\text{tsv}}(\eta, \eta^*) = \exp\bigg[&-|\eta|^2 \left(\sinh^2 r \coth \frac{\beta\omega}{2} + f(\beta) \right) \\ &- \frac{\cosh r \sinh r}{2} \coth \frac{\beta\omega}{2} (e^{-i\varphi}\eta^2 + e^{i\varphi}\eta^{*2}) \bigg]. \end{aligned} \tag{11}$$

Though the space is direct product between the original space and identical copy of it, the observational quantities are the expectation values of a, $a^\dagger$, a^2, $a^{\dagger 2}$ (Ezawa *et al.*, 1991) etc. These quantities can be computed in thermal coherent state, thermal squeezed state, and thermal squeezed vacuum state formalisms by applying their corresponding characteristic function in the following relations,

$$\begin{aligned} \langle a \rangle &= \frac{\partial \mathcal{Q}}{\partial \eta^*}\bigg|_{\eta=\eta^*=0}, \\ \langle a^\dagger \rangle &= -\frac{\partial \mathcal{Q}}{\partial \eta}\bigg|_{\eta=\eta^*=0}. \end{aligned} \tag{12}$$

Similarly the higher order expectation values of a and $a^\dagger$ can also be evaluated using the same procedure of Eq. (12).

3. INFLATON IN A FLAT FRW METRIC

Consider a flat Friedmann–Robertson–Walker spacetime with the line element

$$ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2), \tag{13}$$

the metric is treated as an unquantized external field.

The minimally coupled inflaton with the gravity, for the metric (13), can be described by the Lagrangian

$$L = \frac{1}{2}R^3(\dot{\varphi}^2 - m^2\varphi^2), \tag{14}$$

where overdot represents a derivative with respect to time. The equation governing the inflaton, for the metric (13), can be written as

$$\ddot{\varphi} + 3\frac{\dot{R}}{R}\dot{\varphi} + m^2\varphi = 0. \tag{15}$$

One can define the momentum conjugate to φ as, $\pi = \frac{\partial L}{\partial \dot{\varphi}}$. Thus, the Hamiltonian of the inflaton is

$$H = \frac{\pi^2}{2R^3} + \frac{1}{2}R^3 m^2 \varphi^2. \tag{16}$$

Therefore, $0 - 0$ component of the energy–momentum tensor for the inflaton takes the following form

$$T_{00} = \frac{R^3}{2}(\dot{\varphi}^2 + m^2\varphi^2). \tag{17}$$

Consider the minimally coupled inflaton as the source of gravity. Therefore, the classical Einstein equation becomes

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}\frac{T_{00}}{R^3}, \tag{18}$$

where T_{00} is the energy density of the inflaton, given by (17). In the cosmological context, the classical Einstein equation (18) means that the Hubble constant, $H = \frac{\dot{R}}{R}$, is determined by the energy density of the dynamically evolving inflaton as described by (15).

4. THERMAL AND QUANTUM PARTICLE CREATION

Since there is no consistent quantum theory of gravity available, it would be meaningful to consider the semiclassical gravity theory to study quantum effect of matter field in a classical background metric. The semiclassical approach is also useful to deal with problems in cosmology, where quantum gravity effects are negligible. Oscillatory phase of inflaton is such a situation, where one can neglect the quantum gravity effects. Therefore, the present study can be restricted in the frame work of semiclassical theory of gravity. In semiclassical theory the Einstein equation can be written as

$$G_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle, \tag{19}$$

where the quantum field, represented by a scalar field ϕ, is governed by the time-dependent Schrödinger equation

$$i \frac{\partial \phi}{\partial t} = \hat{H}_\phi \phi. \tag{20}$$

Consider quantum inflaton as the source, then the Friedmann equation, for the metric (13), in the semiclassical theory, can be written as

$$\left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} \frac{1}{R^3} \langle \hat{H}_\varphi \rangle, \tag{21}$$

where $\langle \hat{H}_\varphi \rangle$ represent the expectation value of the Hamiltonian of the inflaton in a quantum state under consideration.

The inflaton can be described by the time dependent harmonic oscillator, with the Hamiltonian (16). To study, the semiclassical Friedmann equation, the expectation value of the Hamiltonian (16) to be computed, in a quantum state under consideration. Therefore (16) becomes

$$\langle \hat{H}_\varphi \rangle = \frac{1}{2R^3} \langle \hat{\pi}^2 \rangle + \frac{m^2 R^3}{2} \langle \hat{\varphi}^2 \rangle. \tag{22}$$

The eigenstates of the Hamiltonian are the Fock states

$$a^\dagger(t) a(t) |n, \varphi, t\rangle = n |n, \varphi, t\rangle, \tag{23}$$

where

$$\begin{aligned} a(t) &= \varphi^*(t)\hat{\pi} - R^3 \dot{\varphi}^*(t)\hat{\varphi}, \\ a^\dagger(t) &= \varphi(t)\hat{\pi} - R^3 \dot{\varphi}(t)\hat{\varphi}. \end{aligned} \tag{24}$$

As an alternative to the n representation, consider the inflaton in thermal squeezed state formalism. Therefore, the expectation value of the Hamiltonian (22) in thermal squeezed state can be computed as follows.

From (9), (12), and (24), we get

$$\begin{aligned} \langle \hat{\pi}^2 \rangle = &-R^6 \dot{\varphi}^2 \left(\alpha^2 - e^{i\vartheta} \cosh r \sinh r \coth \frac{\beta\omega}{2} \right) \\ &- R^6 \dot{\varphi}^{*2} \left(\alpha^{*2} - e^{-i\vartheta} \cosh r \sinh r \coth \frac{\beta\omega}{2} \right) \\ &+ R^6 \dot{\varphi}^* \dot{\varphi} \left(2|\alpha|^2 + 2 \sinh^2 r \coth \frac{\beta\omega}{2} + 2 f(\beta) + 1 \right), \end{aligned} \tag{25}$$

and

$$\langle\hat{\varphi}^2\rangle = -\varphi^2\left(\alpha^2 - e^{i\vartheta}\cosh r\sinh r\coth\frac{\beta\omega}{2}\right)$$
$$-\varphi^{*2}\left(\alpha^{*2} - e^{-i\vartheta}\cosh r\sinh r\coth\frac{\beta\omega}{2}\right)$$
$$+\varphi^*\varphi\left(2|\alpha|^2 + 2\sinh^2 r\coth\frac{\beta\omega}{2} + 2f(\beta) + 1\right). \tag{26}$$

Substituting (25) and (26) in (22), and the apply the result in (21), then the semi-classical Friedmann equation becomes

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{4\pi}{3}\Bigg[(\dot{\varphi}^*\dot{\varphi} + m^2\varphi^*\varphi)\left(2|\alpha|^2 + 2\sinh^2 r\coth\frac{\beta\omega}{2} + 2f(\beta) + 1\right)$$
$$-(\dot{\varphi}^2 + m^2\varphi^2)\left(\alpha^2 - e^{i\vartheta}\cosh r\sinh r\coth\frac{\beta\omega}{2}\right)$$
$$-(\dot{\varphi}^{*2} + m^2\varphi^{*2})\left(\alpha^{*2} - e^{-i\vartheta}\cosh r\sinh r\coth\frac{\beta\omega}{2}\right)\Bigg], \tag{27}$$

where φ and φ^* satisfy Eq. (15) and the Wronskian condition

$$R^3(t)(\dot{\varphi}^*(t)\varphi(t) - \varphi^*(t)\dot{\varphi}(t)) = i. \tag{28}$$

The above boundary condition fixes the normalization constants of the two independent solutions.

To solve the self-consistent semiclassical Einstein equation (27), transform the solution in the following form

$$\varphi(t) = \frac{1}{R^{\frac{3}{2}}}\psi(t), \tag{29}$$

thereby obtaining

$$\ddot{\psi}(t) + \left(m^3 - \frac{3}{4}\left(\frac{\dot{R}(t)}{R(t)}\right)^2 - \frac{3}{2}\frac{\ddot{R}(t)}{R(t)}\right)\psi(t) = 0. \tag{30}$$

Next, focus on the oscillatory phase of the inflaton after inflation. In the parameter region satisfying the inequality

$$m^2 > \frac{3\dot{R}^2}{4R^2} + \frac{2\ddot{R}}{2R}, \tag{31}$$

the inflaton has an oscillatory solution of the form

$$\psi(t) = \frac{1}{\sqrt{2\sigma(t)}}\exp\left(-i\int\sigma(t)dt\right), \tag{32}$$

with

$$\sigma(t) = \sqrt{m^2 - \frac{3}{4}\left(\frac{\dot{R}}{R}\right)^2 - \frac{3}{2}\frac{\ddot{R}}{R} + \frac{3}{4}\left(\frac{\dot{\sigma}(t)}{\sigma(t)}\right)^2 - \frac{1}{2}\frac{\ddot{\sigma}(t)}{\sigma(t)}}. \tag{33}$$

By applying the transform solution (29) in (27), and also using the fact $\alpha = e^{i\delta}\alpha$, we obtain

$$\begin{aligned} R(t) = \Bigg[\frac{2\pi}{3\sigma} \frac{1}{\left(\frac{\dot{R}}{R}\right)^2} \Bigg[\frac{1}{4} &\left(\left(\frac{\dot{R}}{R} + \frac{\dot{\sigma}}{\sigma} \right)^2 + \sigma^2 + m^2 \right) \\ &\times \left(2|\alpha|^2 + 2\sinh^2 r \coth\frac{\beta\omega}{2} + 2f(\beta) + 1 \right) \\ &- \frac{1}{4} \left(\left(3\frac{\dot{R}}{R} + \frac{\dot{\sigma}}{\sigma} \right)^2 - \sigma^2 + m^2 \right) \\ &\times \left(2\alpha^2 \cos(2\delta - 2\sigma t) - \cos(\vartheta - 2\sigma t) \sinh(2r) \coth\frac{\beta\omega}{2} \right) \\ &+ \sigma \left(3\frac{\dot{R}}{R} + \frac{\dot{\sigma}}{\sigma} \right) \\ &\times \left(2\alpha^2 \sin(2\delta - 2\sigma t) + \sin(\vartheta - 2\sigma t) \sinh(2r) \coth\frac{\beta\omega}{2} \right) \Bigg] \Bigg]^{1/3}. \end{aligned} \tag{34}$$

The next order approximation solution of the Eq. (34) can be obtained by using the following approximation ansatzs

$$\sigma_0(t) = m, \tag{35}$$

and

$$R_0(t) = R_0 t^{\frac{2}{3}}. \tag{36}$$

Thus we get

$$\begin{aligned} R_1(t) = \Bigg[3\pi m t^2 \Bigg[&\left(1 + \frac{1}{2m^2t^2} \right) \left(2|\alpha|^2 + 2\sinh^2 r \coth\frac{\beta\omega}{2} + 2f(\beta) + 1 \right) \\ &- \frac{1}{2m^2t^2} \left(2\alpha^2 \cos(2\delta - 2mt) - \cos(\vartheta - 2mt) \sinh(2r) \coth\frac{\beta\omega}{2} \right) \\ &+ \frac{2}{mt^2} \left(2\alpha^2 \sin(2\delta - 2mt) - \sin(\vartheta - 2mt) \sinh(2r) \coth\frac{\beta\omega}{2} \right) \Bigg] \Bigg]^{1/3}. \end{aligned} \tag{37}$$

When $2\delta = \vartheta = 2mt$, then (37) becomes

$$R_1(t) = \left[3\pi mt^2\left[\left(1+\frac{1}{2m^2t^2}\right)\left(2|\alpha|^2 + 2\sinh^2 r\coth\frac{\beta\omega}{2} + 2f(\beta) + 1\right)\right.\right.$$
$$\left.\left. - \frac{1}{2m^2t^2}\left(2\alpha^2 - \sinh(2r)\coth\frac{\beta\omega}{2}\right)\right]\right]^{1/3}. \tag{38}$$

Next, consider the particle production of the inflaton, in thermal squeezed states formalisms in semiclassical theory of gravity. First, consider the Fock space which has a one parameter dependence on the cosmological time t. The number of particles at a later time t produced from the vacuum at the initial time t_0 is given by

$$N_0(t, t_0) = \langle 0, \varphi, t_0|\hat{N}(t)|0, \varphi, t_0\rangle, \tag{39}$$

here, $\hat{N}(t) = a^\dagger a$ and its expectation value can be calculated by using (24). Therefore,

$$\langle\hat{N}(t)\rangle = R^6\dot{\varphi}\dot{\varphi}^*\langle\hat{\varphi}^2\rangle + \varphi\varphi^*\langle\hat{\pi}^2\rangle - R^3\varphi\dot{\varphi}^*\langle\hat{\pi}\hat{\varphi}\rangle - R^3\dot{\varphi}\varphi^*\langle\hat{\varphi}\hat{\pi}\rangle. \tag{40}$$

Again using (24) and (40), we get

$$N_0(t, t_0) = R^6|\varphi(t)\dot{\varphi}(t_0) - \dot{\varphi}(t)\varphi(t_0)|^2. \tag{41}$$

Using (35), (36), and (40), the number of particles created at a later time t from the vacuum state at the initial time t_0 in the limit mt_0, $mt > 1$ can be computed and is given (Kim and Page, 1999) by

$$N_0(t, t_0) = \frac{1}{4\sigma(t)\sigma(t_0)}\left(\frac{R(t)}{R(t_0)}\right)^3$$
$$\times\left[\frac{1}{4}\left(3\frac{\dot{R}(t)}{R(t)} - 3\frac{\dot{R}(t_0)}{R(t_0)} - \frac{\dot{\sigma}(t)}{\sigma(t)} + \frac{\dot{\sigma}(t_0)}{\sigma(t_0)}\right)^2 + (\sigma(t) - \sigma(t_0))^2\right]$$
$$\simeq \frac{(t-t_0)^2}{4m^2t_0^4}. \tag{42}$$

To compute the particle creation in thermal squeezed state, the expectation values of the $\langle\hat{\pi}^2\rangle$, $\langle\hat{\varphi}^2\rangle$, $\langle\hat{\pi}\hat{\varphi}\rangle$, and $\langle\hat{\varphi}\hat{\pi}\rangle$ in the thermal squeezed state are required. Thus using Eqs. (9), (12), (24), and (40), we get

$$N_{\text{tss}}(t, t_0) = \frac{1}{16}\frac{1}{\sigma(t)}\frac{1}{\sigma(t_0)}\left(\frac{R(t)}{R(t_0)}\right)^3$$
$$\times\left[\left[\left(3\frac{\dot{R}(t)}{R(t)} - 3\frac{\dot{R}(t_0)}{R(t_0)} + \frac{\dot{\sigma}(t)}{\sigma(t)} - \frac{\dot{\sigma}(t_0)}{\sigma(t_0)}\right)^2 + \sigma(t)^2 - \sigma(t_0)^2\right]\right.$$

$$\times \left(2|\alpha|^2 + 2\sinh^2 r \coth\frac{\beta\omega}{2} + 2f(\beta) + 1\right)$$
$$- \left(3\frac{\dot{R}(t)}{R(t)} - 3\frac{\dot{R}(t_0)}{R(t_0)} + \frac{\dot{\sigma}(t)}{\sigma(t)} - \frac{\dot{\sigma}(t_0)}{\sigma(t_0)}\right)^2$$
$$\times \left[\left(\alpha^2 - e^{i\vartheta}\cosh r \sinh r \coth\frac{\beta\omega}{2}\right)e^{2i\sigma(t_0)t_0}\right.$$
$$\left. + \left(\alpha^{*2} - e^{-i\vartheta}\cosh r \sinh r \coth\frac{\beta\omega}{2}\right)e^{2i\sigma(t_0)t_0}\right]\Bigg]. \tag{43}$$

Which is the number of particles produced in thermal squeezed state, at a later time t from the initial time t_0.

By taking $\alpha = e^{i\delta}\alpha$, and using (35) and (36), Eq. (43) becomes

$$N_{\text{tss}} \simeq N_0(t, t_0)\left[2|\alpha|^2 + 2\sinh^2 r \coth\frac{\beta\omega}{2} + 2f(\beta) + 1\right.$$
$$\left. - 2\alpha^2\cos(2\delta - 2mt_0) + \cos(\vartheta - 2mt_0)\sinh(2r)\coth\frac{\beta\omega}{2}\right], \tag{44}$$

where $N_0(t, t_0)$ is given by (42).

When $r = 0$, Eq. (44) leads to

$$N_{\text{tcs}} \simeq N_0(t, t_0)[2|\alpha|^2 + 2f(\beta) + 1 - 2\alpha^2\cos(2\delta - 2mt_0)]. \tag{45}$$

Which is particle creation in thermal coherent state. The same result can be also obtained by using Eqs. (5), (12), (24), (35), (36), and (40).

When $\alpha = 0$, Eq. (44) becomes

$$N_{\text{tsv}} \simeq N_0(t, t_0)\left[2\sinh^2 r \coth\frac{\beta\omega}{2} + 2f(\beta) + 1\right.$$
$$\left. + \cos(\vartheta - 2mt_0)\sinh(2r)\coth\frac{\beta\omega}{2}\right]. \tag{46}$$

Equation (46) can be also obtained by using Eqs. (11), (12), (24), (35), (36), and (40), and is the particle production due thermal squeezed vacuum state.

When $2\delta = 2mt_0$ and $\vartheta = 2mt_0$, Eqs. (44), (45), and (46), respectively become

$$N_{\text{tss}} \simeq N_0(t, t_0)\left[2|\alpha|^2 + 2\sinh^2 r \coth\frac{\beta\omega}{2} + 2f(\beta) + 1\right.$$
$$\left. - 2\alpha^2 + \sinh(2r)\coth\frac{\beta\omega}{2}\right], \tag{47}$$

$$N_{\text{tcs}} \simeq N_0(t, t_0)[2|\alpha|^2 + 2f(\beta) + 1 - 2\alpha^2], \tag{48}$$

and

$$N_{\text{tsv}} \simeq N_0(t, t_0)\left[2\sinh^2 r + 2f(\beta) + 1\sinh(2r)\coth\frac{\beta\omega}{2}\right]. \tag{49}$$

When $r = \alpha = 0$, then Eq. (44) takes the following form

$$N_{\text{th}} \simeq N_0(t, t_0)[2f(\beta) + 1]. \tag{50}$$

Which is the particle creation due to purely thermal effects.

5. CONCLUSIONS

In this paper, we studied particle production of the coherently oscillating inflaton, after the inflation, in thermal coherent states and thermal squeezed states formalisms, in the frame work of semiclassical theory of gravity. The number of particles at a later time t, produced from the thermal coherent state, at the initial time t_0, in the limit $mt_0, mt > 1$ calculated. It shows, the particle production depends on the coherent state parameter and finite temperature effects. The particle creation in thermal squeezed vacuum state in the limit $mt_0 > mt > 1$ is also computed, it is found that the particle production depending on the associated squeezing parameter and temperature. Similarly the number of particles produced in thermal squeezed state also computed. It is observed that, when $r = 0$, the result agree with the number of particles produced in the thermal coherent state and when $\alpha = 0$, the result is equal to the number of particles created in thermal squeezed vacuum state.

The approximate leading solution obtained for the Einstein equation, in the thermal squeezed states shows oscillatory behavior except when the condition, $2\delta = \vartheta = 2mt$, satisfies. Though both classical and quantum inflaton in the oscillatory phase of the inflaton lead the same power-law expansion, the correction to the expansion does not show any oscillatory behavior in semiclassical gravity in contrast to the oscillatory behavior seen in classical gravity only when $2\delta = \vartheta = 2mt$. It is also noted that, the coherently oscillating inflaton, in thermal squeezed vacuum, thermal squeezed, and thermal coherent states representation, suffer particle creation and created particle exhibit oscillations. The oscillation of the created particles is necessary to preheat the universe to hot again after the inflation. The present study can account for the postinflation particle creation due to thermal and quantum effects of inflaton in a flat FRW universe. Since the created particles oscillate, we hope that this kind of study can throw light on preheating issues of postinflationary scenario.

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